

1 **CLAIMS**

2 What is claimed is:

3 1. A method comprising:

4 determining at least one Squared Tate pairing for at least one hyperelliptic
5 curve; and

6 cryptographically processing selected information based on said determined
7 Squared Tate pairing.

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9 2. The method as recited in Claim 1, wherein said Squared Tate pairing
10 is defined for at least one hyperelliptic curve C of genus g over a field K .

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12 3. The method as recited in Claim 1, wherein determining said Squared
13 Tate pairing further includes:

14 forming a mathematical chain for m , wherein m is a positive integer and an
15 m -torsion element D is fixed on Jacobian of said hyperelliptic curve C .

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17 4. The method as recited in Claim 3, wherein said mathematical chain
18 includes a mathematical chain selected from a group of mathematical chains
19 comprising an addition chain and an addition-subtraction chain.

1 5. A computer-readable medium having computer-implementable
2 instructions for causing at least one processing unit to perform acts comprising:

3 calculating at least one Squared Tate pairing for at least one hyperelliptic
4 curve; and

5 cryptographically processing selected information based on said determined
6 Squared Tate pairing.

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8 6. The computer-readable medium as recited in Claim 5, wherein said
9 Squared Tate pairing is defined for at least one hyperelliptic curve C of genus g
10 over a field K .

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12 7. The computer-readable medium as recited in Claim 5, wherein
13 determining said Squared Tate pairing further includes:

14 forming a mathematical chain for m , wherein m is a positive integer and an
15 m -torsion element D is fixed on Jacobian of said hyperelliptic curve C .

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17 8. The computer-readable medium as recited in Claim 7, wherein said
18 mathematical chain includes a mathematical chain selected from a group of
19 mathematical chains comprising an addition chain and an addition-subtraction
20 chain.

1 9. An apparatus comprising:

2 memory configured to store information suitable for use with using a
3 cryptographic process;

4 logic operatively coupled to said memory and configured to calculate at
5 least one Squared Tate pairing for at least one hyperelliptic curve, and at least
6 partially support cryptographic processing of selected stored information based on
7 said determined Squared Tate pairing.

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9 10. The apparatus as recited in Claim 9, wherein said Squared Tate
10 pairing is defined for at least one hyperelliptic curve C of genus g over a field K .

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12 11. The apparatus as recited in Claim 9, wherein said logic is further
13 configured to form a mathematical chain for m , wherein m is a positive integer and
14 an m -torsion element D is fixed on Jacobian of said hyperelliptic curve C .

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16 12. The apparatus as recited in Claim 11, wherein said mathematical
17 chain includes a mathematical chain selected from a group of mathematical chains
18 comprising an addition chain and an addition-subtraction chain.

1 13. A method comprising:

2 determining a hyperelliptic curve C of genus g over a field K and a positive
3 integer m ;

4 determining a Jacobian $J(C)$ of said hyperelliptic curve C , and wherein each
5 element D of $J(C)$ contains a representative of the form $A - g(\mathbf{P}_0)$, where A is an
6 effective divisor of degree g ; and

7 determining a plurality of functions $h_{j,D}$ that are iterative building blocks for
8 the formation of a function $h_{m,D}$ in order to evaluate v_m which is a Squared Tate
9 pairing.

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11 14. The method as recited in Claim 13, wherein said hyperelliptic curve
12 C is over a field not of characteristic 2.

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14 15. The method as recited in Claim 13, wherein

15 for at least one element D of $J(C)$, a representative for iD will be $A_i - g(\mathbf{P}_0)$,
16 where A_i is effective of degree g .

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18 16. The method as recited in Claim 13, wherein if $P=(x, y)$ is a point on
19 said hyperelliptic curve C , then $-\mathbf{P}$ denotes a point $-\mathbf{P}:=(x, -y)$, and wherein if a
20 point $P=(x, y)$ occurs in A and $y \neq 0$, then $-\mathbf{P} := (x, -y)$ does not occur in A and a
21 representative for identity will be $g(\mathbf{P}_0)$.

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23 17. The method as recited in Claim 16, further comprising:

24 to a representative A_i , associating two polynomials (a_i, b_i) which represent a
25 divisor.

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2 18. The method as recited in Claim 16, further comprising:

3 determining D as an m -torsion element of $J(C)$.

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5 19. The method as recited in Claim 18, further comprising:

6 if j is an integer, then $h_{j,D} = h_{j,D}(X)$ denoting a rational function on C with
7 divisor $(h_{j,D}) = jA_1 - A_j - ((j-1)g)(\mathbf{P}_0)$.

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9 20. The method as recited in Claim 18, wherein D is an m -torsion
10 divisor and $A_m = g(\mathbf{P}_0)$, and a divisor of $h_{m,D}$ is $(h_{m,D}) = mA_1 - mg(\mathbf{P}_0)$.

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12 21 The method as recited in Claim 18, wherein $h_{m,D}$ is well-defined up
13 to a multiplicative constant.

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15 22. The method as recited in Claim 18, further comprising:
16 evaluating $h_{m,D}$ at a degree zero divisor E on said hyperelliptic curve C ,
17 wherein E does not contain \mathbf{P}_0 and E is prime to A_i .

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19 23. The method as recited in Claim 18, wherein E is prime to A_i for all i
20 in an addition-subtraction chain for m .

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22 24. The method as recited in Claim 22, wherein given A_i , A_j , and A_{i+j} ,
23 further comprising determining a function $u_{i,j}$ such that a divisor of $u_{i,j}$ is $(u_{i,j}) = A_i$
24 + $A_j - A_{i+j} - g(\mathbf{P}_0)$.

1 25. The method as recited in Claim 22, further comprising:

2 evaluating $h_{j,D}(E)$ such that when $j=1$, $h_{1,D}$ is 1.

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4 26. The method as recited in Claim 22, further comprising:

5 given A_i , A_j , $h_{i,D}(E)$ and $h_{j,D}(E)$, evaluating $u_{i,j}$ to be $(u_{i,j}) = A_i + A_j - A_{i+j} -$
6 $g(\mathbf{P}_0)$, and $h_{i+j,D}(E) = h_{i,D}(E) h_{j,D}(E) u_{i,j}(E)$.

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8 27. The method as recited in Claim 13, further comprising:

9 determining a function $(u_{i,j}) = A_i + A_j - A_{i+j} - g(\mathbf{P}_0)$.

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11 12 28. The method as recited in Claim 27, wherein $g = 2$ and

13 14 $(u_{i,j}) = A_i + A_j - A_{i+j} - 2(\mathbf{P}_0)$ is determined as follows

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$$u_{i,j}(\mathbf{X}) := \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(\mathbf{X}))$$
, if the degree of a_{new} is

17 greater than 2, otherwise, $u_{i,j}$ is determined as $u_{i,j}(\mathbf{X}) := d(x(\mathbf{X}))$, wherein
18 $d(x)$ is the greatest common divisor of three polynomials $(a_i(x), a_j(x), b_i(x) + b_j(x))$.

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20 21 29. The method as recited in Claim 13, further comprising:

22 determining a Squared Tate pairing for a hyperelliptic curves v_m , for an m -
23 torsion element D of a Jacobian $J(C)$ and an element E of $J(C)$, with
24 representatives $(\mathbf{P}_1) + (\mathbf{P}_2) + \dots + (\mathbf{P}_g) - g(\mathbf{P}_0)$ and $(\mathbf{Q}_1) + (\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - g(\mathbf{P}_0)$,

1 respectively, with each \mathbf{P}_i and each \mathbf{Q}_j on the curve C , with \mathbf{P}_i not equal to $\pm\mathbf{Q}_j$
2 for all i, j , determining that

3 $v_m(D, E) := (h_{m,D}((\mathbf{Q}_1) - (-\mathbf{Q}_1) + (\mathbf{Q}_2) - (-\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - (-\mathbf{Q}_g)))^{\frac{q-1}{m}}$.

4
5 30. A computer-readable medium having computer-implementable
6 instructions for causing at least one processing unit to perform acts comprising:

7 determining a hyperelliptic curve C of genus g over a field K and a positive
8 integer m ;

9 determining a Jacobian $J(C)$ of said hyperelliptic curve C , and wherein each
10 element D of $J(C)$ contains a representative of the form $A - g(\mathbf{P}_0)$, where A is an
11 effective divisor of degree g ; and

12 determining a plurality of functions $h_{j,D}$ that are iterative building blocks for
13 the formation of a function $h_{m,D}$ in order to evaluate v_m which is a Squared Tate
14 pairing.

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16 31. The computer-readable medium as recited in Claim 30, wherein said
17 hyperelliptic curve C is not of characteristic 2.

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19 32. The computer-readable medium as recited in Claim 30, wherein
20 for at least one element D of $J(C)$, a representative for iD will be $A_i - g(\mathbf{P}_0)$,
21 where A_i is effective of degree g .

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23 33. The computer-readable medium as recited in Claim 30, wherein if
24 $P = (x, y)$ is a point on said hyperelliptic curve C , then $-\mathbf{P}$ denotes a point $-\mathbf{P} := (x,$

1 $-y$), and wherein if a point $P=(x, y)$ occurs in A and $y \neq 0$, then $\neg P := (x, -y)$ does
2 not occur in A and a representative for identity will be $g(\mathbf{P}_0)$.

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4 34. The computer-readable medium as recited in Claim 33, further
5 comprising:

6 to a representative A_i , associating two polynomials (a_i, b_i) which represent a
7 divisor.

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9 35. The computer-readable medium as recited in Claim 33, further
10 comprising:

11 determining D as an m -torsion element of $J(C)$.

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13 36. The computer-readable medium as recited in Claim 35, further
14 comprising:

15 if j is an integer, then $h_{j,D} = h_{j,D}(X)$ denoting a rational function on C with
16 divisor $(h_{j,D}) = jA_1 - A_j - ((j-1) g)(\mathbf{P}_0)$.

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18 37. The computer-readable medium as recited in Claim 35, wherein D is
19 an m -torsion divisor and $A_m = g(\mathbf{P}_0)$, and a divisor of $h_{m,D}$ is $(h_{m,D}) = mA_1 - mg(\mathbf{P}_0)$.

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21 38 The computer-readable medium as recited in Claim 35, wherein $h_{m,D}$
22 is well-defined up to a multiplicative constant.

1 39. The computer-readable medium as recited in Claim 35, further
2 comprising:

3 evaluating $h_{m,D}$ at a degree zero divisor E on said hyperelliptic curve C ,
4 wherein E does not contain P_0 and E is prime to A_i .

5
6 40. The computer-readable medium as recited in Claim 35, wherein E is
7 prime to A_i for all i in an addition-subtraction chain for m .

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9 41. The computer-readable medium as recited in Claim 39, wherein
10 given A_i , A_j , and A_{i+j} , further comprising determining a function $u_{i,j}$ such that a
11 divisor of $u_{i,j}$ is $(u_{i,j}) = A_i + A_j - A_{i+j} - g(\mathbf{P}_0)$.

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13 42. The computer-readable medium as recited in Claim 39, further
14 comprising:

15 evaluating $h_{j,D}(E)$ such that when $j=1$, $h_{1,D}$ is 1.

16
17 43. The computer-readable medium as recited in Claim 39, further
18 comprising:

19 given A_i , A_j , $h_{i,D}(E)$ and $h_{j,D}(E)$, evaluating $u_{i,j}$ to be $(u_{i,j}) = A_i + A_j - A_{i+j} -$
20 $g(\mathbf{P}_0)$, and $h_{i+j,D}(E) = h_{i,D}(E) h_{j,D}(E) u_{i,j}(E)$.

21
22 44. The computer-readable medium as recited in Claim 30, further
23 comprising:

24 determining a function $(u_{i,j}) = A_i + A_j - A_{i+j} - g(\mathbf{P}_0)$.
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1 45. The computer-readable medium as recited in Claim 44, wherein $g =$
2 2 and

3 $(u_{i,j}) = A_i + A_j - A_{i+j} - 2(\mathbf{P}_0)$ is determined as follows

4 $u_{i,j}(\mathbf{X}) := \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(\mathbf{X}))$, if the degree of a_{new} is

5 greater than 2, otherwise, $u_{i,j}$ is determined as $u_{i,j}(\mathbf{X}) := d(x(\mathbf{X}))$, wherein
6 $d(x)$ is the greatest common divisor of three polynomials $(a_i(x), a_j(x), b_i(x) + b_j(x))$.

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10 46. The computer-readable medium as recited in Claim 30, further
11 comprising:

12 determining a Squared Tate pairing for a hyperelliptic curves v_m , for an m -
13 torsion element D of a Jacobian $J(C)$ and an element E of $J(C)$, with
14 representatives $(\mathbf{P}_1) + (\mathbf{P}_2) + \dots + (\mathbf{P}_g) = g(\mathbf{P}_0)$ and $(\mathbf{Q}_1) + (\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) = g(\mathbf{P}_0)$,
15 respectively, with each \mathbf{P}_i and each \mathbf{Q}_j on the curve C , with \mathbf{P}_i not equal to $\pm \mathbf{Q}_j$
16 for all i, j , determining that

17 $v_m(D, E) := (h_{m,D}((\mathbf{Q}_1) - (-\mathbf{Q}_1) + (\mathbf{Q}_2) - (-\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - (-\mathbf{Q}_g)))^{\frac{q-1}{m}}$.

1 47. An apparatus comprising:

2 memory configured to store information suitable for use with using a
3 cryptographic process; and

4 logic operatively coupled to said memory and configured to determine a
5 hyperelliptic curve C of genus g over a field K and a positive integer m , determine
6 a Jacobian $J(C)$ of said hyperelliptic curve C , wherein each element D of $J(C)$
7 contains a representative of the form $A - g(\mathbf{P}_0)$ and A is an effective divisor of
8 degree g , and determine a plurality of functions $h_{j,D}$ that are iterative building
9 blocks for the formation of a function $h_{m,D}$ in order to evaluate v_m which is a
10 Squared Tate pairing.

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12 48. The apparatus as recited in Claim 47, wherein said hyperelliptic
13 curve C is not of characteristic 2.

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15 49. The apparatus as recited in Claim 47, wherein
16 for at least one element D of $J(C)$, a representative for iD will be $A_i - g(\mathbf{P}_0)$,
17 where A_i is effective of degree g .

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19 50. The apparatus as recited in Claim 47, wherein if $P=(x, y)$ is a point
20 on said hyperelliptic curve C , then $-\mathbf{P}$ denotes a point $-\mathbf{P}:=(x, -y)$, and wherein if
21 a point $P=(x, y)$ occurs in A and $y \neq 0$, then $-\mathbf{P} := (x, -y)$ does not occur in A and a
22 representative for identity will be $g(\mathbf{P}_0)$.

1 51. The apparatus as recited in Claim 50, wherein said logic is further
2 configured to, for a representative A_i , associate two polynomials (a_i, b_i) which
3 represent a divisor.

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5 52. The apparatus as recited in Claim 50, wherein said logic is further
6 configured to determine D as an m -torsion element of $J(C)$.

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8 53. The apparatus as recited in Claim 52, wherein said logic is further
9 configured to, if j is an integer, then determine $h_{j,D} = h_{j,D}(X)$ by denoting a rational
10 function on C with divisor $(h_{j,D}) = jA_1 - A_j - ((j-1) g)(\mathbf{P}_0)$.

11

12 54. The computer-readable medium as recited in Claim 52, wherein D is
13 an m -torsion divisor and $A_m = g(\mathbf{P}_0)$, and a divisor of $h_{m,D}$ is $(h_{m,D}) = mA_1 - mg(\mathbf{P}_0)$.

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15 55. The apparatus as recited in Claim 52, wherein $h_{m,D}$ is well-defined
16 up to a multiplicative constant.

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18 56. The apparatus as recited in Claim 52, wherein said logic is further
19 configured to evaluate $h_{m,D}$ at a degree zero divisor E on said hyperelliptic curve
20 C , wherein E does not contain \mathbf{P}_0 and E is prime to A_i .

21

22 57. The apparatus as recited in Claim 52, wherein E is prime to A_i for all
23 i in an addition-subtraction chain for m .

1 58. The apparatus as recited in Claim 56, wherein given A_i , A_j , and A_{i+j} ,
2 and wherein said logic is further configured to determine a function $u_{i,j}$ such that a
3 divisor of $u_{i,j}$ is $(u_{i,j}) = A_i + A_j - A_{i+j} - g(\mathbf{P}_0)$.

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5 59. The apparatus as recited in Claim 56, wherein said logic is further
6 configured to evaluate $h_{j,D}(E)$ such that when $j=1$, $h_{1,D}$ is 1.

7

8 60. The apparatus as recited in Claim 56, wherein said logic is further
9 configured to, given A_i , A_j , $h_{i,D}(E)$ and $h_{j,D}(E)$, evaluate $u_{i,j}$ to be $(u_{i,j})=A_i + A_j - A_{i+j}$
10 $- g(\mathbf{P}_0)$, and $h_{i+j,D}(E)= h_{i,D}(E) \ h_{j,D}(E) \ u_{i,j} (E)$.

11

12 61. The apparatus as recited in Claim 47, wherein said logic is further
13 configured to determine a function $(u_{i,j}) = A_i + A_j - A_{i+j} - g(\mathbf{P}_0)$.

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15 62. The apparatus as recited in Claim 61, wherein $g = 2$ and
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17 $(u_{i,j}) = A_i + A_j - A_{i+j} - 2(\mathbf{P}_0)$ is determined by said logic as follows

18
$$u_{i,j}(\mathbf{X}) := \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(\mathbf{X}))$$
, if the degree of a_{new} is
19

20 greater than 2, otherwise, $u_{i,j}$ is determined as $u_{i,j}(\mathbf{X}) := d(x(\mathbf{X}))$, wherein $d(x)$ is
21 the greatest common divisor of three polynomials $(a_i(x), a_j(x), b_i(x)+b_j(x))$.

1 63. The apparatus as recited in Claim 47, wherein said logic is further
2 configured to determine a Squared Tate pairing for a hyperelliptic curves v_m , for an
3 m -torsion element D of a Jacobian $J(C)$ and an element E of $J(C)$, with
4 representatives $(\mathbf{P}_1) + (\mathbf{P}_2) + \dots + (\mathbf{P}_g) = g(\mathbf{P}_0)$ and $(\mathbf{Q}_1) + (\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) = g(\mathbf{P}_0)$,
5 respectively, with each \mathbf{P}_i and each \mathbf{Q}_j on the curve C , with \mathbf{P}_i not equal to $\pm \mathbf{Q}_j$
6 for all i, j , and to determine that

7 $v_m(D, E) := (h_{m,D}((\mathbf{Q}_1) - (-\mathbf{Q}_1) + (\mathbf{Q}_2) - (-\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - (-\mathbf{Q}_g)))^{\frac{q-1}{m}}.$

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